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St. Petersburg paradox

A coin game whose expected value is infinite, although no sensible person would pay more than a few coins to play it.

How it works

1. The prize doubles with each toss before heads, while the chance of reaching that toss halves.
2. So every possible outcome adds half a ducat to the game's expected value.
3. The possible outcomes never end, so the expected value of the game is infinite.
4. The infinite value comes only from huge, rare prizes that no bank could ever pay.

Example

Peter and Paul's coin game, Bernoulli 1738

- Half of all games end on the first toss with a prize of 1 ducat.
- The chance of winning more than 20 ducats is only 1 in 32.

→ Bernoulli: the game is worth about 6 ducats to a player who owns 1,000 ducats.

→ Even a bank with a trillion ducats could offer the game fairly for about 21 ducats.

Use it for

- ✓ Deciding whether insurance that costs more than the expected loss is still worth buying
- ✓ Refusing a bet with a good average if losing it would cost the rent
- ✓ Judging a lottery or start-up pitch that rests on a rare, enormous payoff

Watch out

- ✗ Bernoulli's utility answer fails for faster-growing prizes, which bring the infinite value back.
- ✗ The paradox does not make expected value useless; it works for small, repeated bets.

Remember Ask what usually happens and whether anyone could pay the biggest prize.